

# Piggyback whorls: A new theoretical morphologic model reveals constructional linkages among morphological characters in ammonoids

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A new theoretical morphological model is proposed for the analysis of growth, form and morphospace of ammonoid shells. In this model, the shape of a radial cross section through the shell is simulated by “piggybacking” of successive whorls. The “piggyback whorls model” is defined in terms of the enlarging rate of the perimeter and the proportion of the dorsal wall to the whorl periphery, if an isometric relationship is assumed between perimeter and area of the cross-sectioned whorl. Allometric coefficients on these growth parameters determine how compressed and evolute shells are formed. The present model successfully reproduced some correlations among purely geometric variables that have been reported in previous works and were also observed in our biometric analyses. This model yields a hypothesis of “constructional linkages” between aperture shape and coiling geometry that might provide a functional coupling between hydrostatic and hydrodynamic characters. The model may partly explain Buckman’s Law of Covariation between rib features and shell shapes.

Key words: Theoretical morphology, ammonoids, cross-sectional shape, morphospace, constructional linkage, morphological diversity.

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## Introduction

Ammonoids have undoubtedly been the most popular targets for morphospace analysis among fossil animals. Since Raup’s (1967) pioneer work, large collections of morphological data have been used to explore functional morphology, macro-evolutionary trend, and/or disparity of normally coiled ammonoids (Ward 1980; Bayer and McGhee 1984; Saunders and Swan 1984; Swan and Saunders 1987; Nikolaeva and Barskov 1994; Dommergues et al. 1996; Saunders and Work, 1996, 1997; Korn 2000; Korn and Klug 2003; McGowan 2004; Saunders et al. 2004; Gottobrio and Saunders 2005). A theoretical morphological model introduced by Raup (1967) has been widely applied to investigation of ammonoid morphospace, in which gross shell geometries are defined by whorl expansion rate ( $W$ ), width of umbilicus ( $D$ ) and whorl shape ( $S$ ). The occupation patterns of the  $W$ - $D$ - $S$  morphospace are

commonly used for assessing diversity in basic shell geometry. In addition, Saunders and Swan (1984) and Saunders and Work (1996) introduced several different parameters representing sculpture, aperture shape and suture complexity. The Raup’s  $W$ ,  $D$ ,  $S$ , and other metric parameters used previously are suitable for non-destructive morphometric analysis of museum collections, and also available for measurements from photographs in publications. The use of these accessible parameters allows the development of a comprehensive database that includes hundreds of ammonoid taxa. Comparison of these parameters has proven to be a fruitful way for surveying the long-term evolutionary history of morphological diversity of ammonoids (Saunders and Swan 1984; Swan and Saunders 1987; Korn and Klug 2003).

However, previous morphospace analyses were based on data sets collected from adult or submature average individuals and did not deal with ontogenetic change of shell shape, al-

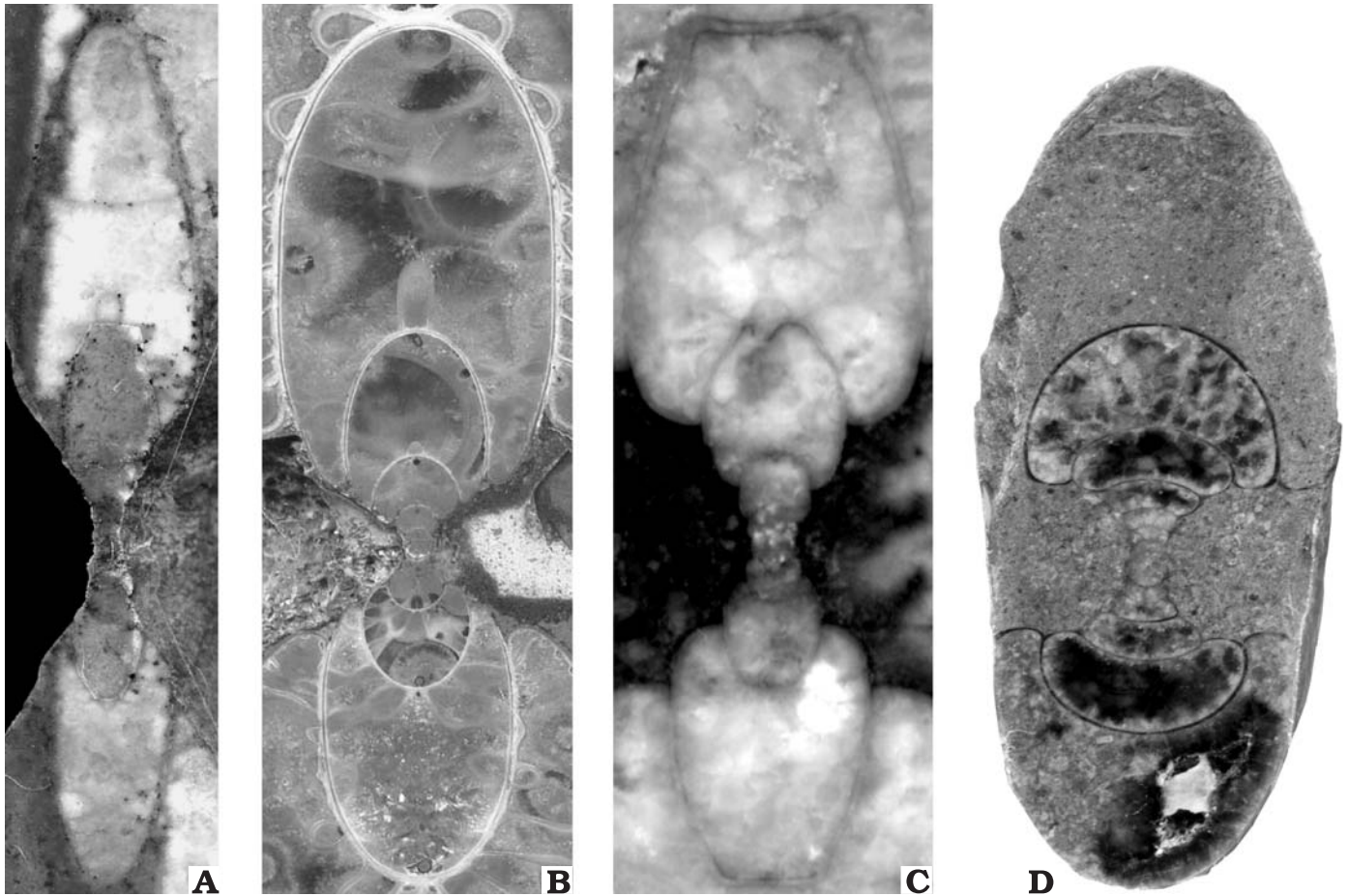


Fig. 1. Radial cross sections of ammonoids to illustrate allometric growth. **A.** *Beloceras* sp., Devonian; Erfoud, Morocco. **B.** *Phylloceras consanguineum* Gemmellaro, 1876, Jurassic; Sakaraha, Madagascar. **C.** *Meekoceras gracilitatis* White, 1879, Triassic; Crittenden Spring, Nevada. **D.** *Girtyoceras meslerianum* (Girty, 1909), Carboniferous; Jackforth Creek, Oklahoma.

though allometry is common in ammonoids (Fig. 1; e.g., Jacobs 1992; Korn and Klug 2002). Thus, ontogenetic aspects have been incorporated tenuously in morphospace analyses of ammonoids (see Okamoto 1996). Ontogeny can not be fully understood in terms of the models that simply specify the gross geometry of shell coiling and aperture shape. Better understanding of the relationship between ontogenetic aspects and morphological diversity requires focusing on the geometric balance between size parameters that determines growth direction (Ubukata 2002). A promising approach for connecting the gross shell geometry with specific growth processes may lie through analysis of the profile of the band of newly accreted material around the aperture that is characterized as the “aperture map” (Rice 1998; Ubukata 2003) or a set of aperture growth vectors regulating shell coiling (Hammer and Bucher 2005b). However, a morphospace provided by the aperture map model is necessarily high-dimensional and is not practical for morphospace analysis.

This paper introduces a new theoretical model for shell growth of normally coiled ammonoids and applies it to morphospace analysis. This model is designed to simulate allometric growth of a cross-sectioned ammonoid shell. The basic assumption of the model is isometric relationship be-

tween the circumference and area of the cross-sectioned whorl; shape of the whorl section is passively regulated so as to conform the assumption. To verify the assumptions is the primary objective of this study. For this purpose, the occupation patterns of morphospace were analyzed in more than 100 Devonian to Cretaceous ammonoid species.

*Institutional abbreviation.*—UMUT, University Museum, University of Tokyo, Japan.

## Modeling of cross-sectioned ammonoid shells

**Background and basic concept.**—The conventional model introduced by Raup (1967) represents growth of a planispiral shell as an expansion and rotation of a generating curve, which approximates the shape of the aperture, around a fixed coiling axis. Subsequently, Okamoto (1988) and Ackerly (1989) introduced “moving frame models” in which the amount and direction of shell growth is defined with respect to the previous aperture. Both methods conceive a shell as

the trajectory of successive generating curves that retain their shapes throughout growth. More recently, Stone (1995) proposed a “CerioShell model” defined by different expansion rates of the aperture in the horizontal and vertical dimensions for visualizing allometric change of the aperture shape and geometry of coiling. The shape of the aperture can also change if helicospiral growth is assumed for each point on the aperture (Bayer 1978; McGhee 1978; Savazzi 1987; Checa 1991; Checa and Aguado 1992) or multiple growth vectors installed around the previous aperture determine the shape of the next aperture (Hammer and Bucher 2005b).

In these previous models, no association was explicitly defined between the shape of the newly formed aperture and form of the preceding whorl. However, in normally coiled ammonoids, adjacent whorls overlap, and the soft part which formed the succeeding whorl must have covered the preceding whorl from outside. The outer component of the dorsal wall of the succeeding whorl is regarded as a product of organic secretion at the suprachephalic mantle fold (Kulicki et al. 2001). Therefore, not only the shape of the previous aperture but also the form of the ventral surface of the preceding whorl should influence geometry of the newly formed aperture around the mantle margin. Hutchinson (1989) proposed a feedback mechanism in which the preceding whorl provides a “road” for the mantle margin and guides shell growth. His “road-holding model” may explain how a succeeding whorl maintains its position with respect to the preceding whorl to achieve planispiral geometry, but is not concerned with how to regulate the whorl shape. Morita (1991a, b) simulated behavior of an expanding elastic mantle represented by a finite element model of a double membrane tube. His “DMS-tube model” predicted that an expanding mantle tends to be elongated ventrally if the dorsal part of the mantle is fixed on the shell. Such mantle behavior may be a possible mechanism to regulate the whorl shape of gastropods in which the head-foot mass presses the mantle margin near the coiling axis when the snail clamps its body to the hard bottom (Morita 2003). However, this mechanism does not seem applicable to morphogenesis of nektonic ammonoids.

In the present study, we attempt to realize the process of piling up of the ammonoid whorls in which the shape of each whorl is determined in relation to the form of the preceding whorl. Therefore, the model introduced herein is based on a “moving reference frame” taking into account previous whorls and the present approach does not need the coiling axis except for measurements. The mode of accretionary growth seems to be influenced by the geometric balance of growth kinematics around the shell margin (Ubukata 2002). However, it is not easy to measure parameters from actual specimens defined by a three-dimensional model including these algorithms or developmental processes. Here, to simplify the problem, we developed a geometric model designed to simulate two-dimensional patterns which correspond to the radial cross sections of ammonoid shells. Since growth lines of ammonoid shells are not always linear or rectidi-

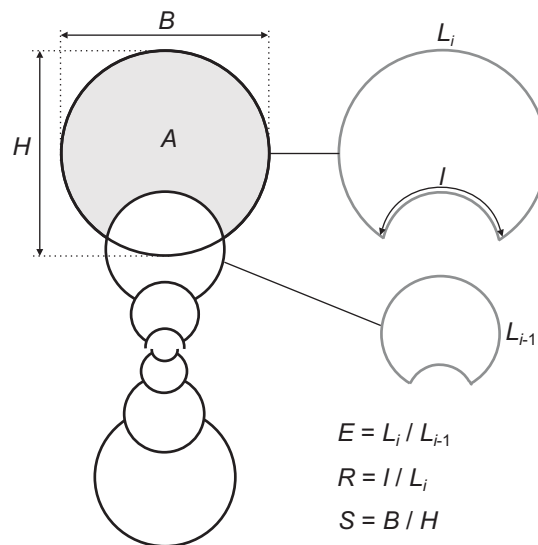


Fig. 2. Schematic figure of a cross section of the ammonoid shell showing how to define parameters of the piggyback whorls model. The enlarging rate of the whorl periphery ( $E$ ) is defined as the ratio of the total perimeter of the newly added whorl section ( $L_i$ ) to that of preceding whorl ( $L_{i-1}$ ). The proportion of the dorsal whorl ( $R$ ) is given by the ratio of the circumferential length along the dorsal wall ( $I$ ) with respect to  $L_i$ . The shape of the whorl section ( $S$ ) is represented by the ratio of breadth ( $B$ ) and height ( $H$ ) of the whorl.

radiate, a radial cross section of each whorl does not exactly represent the aperture shape but is a rough approximation of a slice of newly added shell material. Usefulness of the present geometric model can be assessed if results of computer simulations are compared with biometric data.

**Piggyback whorls model.**—In the cross section, growth of a normally coiled ammonoid shell is represented by additions of successive whorls: a successive whorl “piggybacks” on the immediately preceding one. Geometric properties of the cross-sectioned shell consist of size, shape and positional relationship between neighboring whorls. In a simple case, the cross section of the whorl is regarded as elliptic (Fig. 2). Since shelly material is secreted at the distal extremity of mantle, increase rate of the perimeter of the whorl stands for the growth rate of the mantle. If the size of each whorl is defined by the perimeter of the whorl, the growth rate in the radial direction is determined by the enlarging ratio of the perimeter with respect to the preceding whorl,  $E$ . That is,

$$L_i = EL_{i-1} \quad (1)$$

where  $L_i$  and  $L_{i-1}$  are total perimeters of the whorl section of the succeeding and preceding whorls, respectively. Also the growth rate of body mass is reflected by the enlarging rate of the area of the cross-sectioned whorl. The ratio of the area to the perimeter influences the shape of the whorl section which is defined by the aspect ratio of the ellipse,  $S$  (see Fig. 2). In the present model, the area<sup>0.5</sup>/perimeter ratio is assumed to be constant throughout growth as mentioned below. The positional relationship between neighboring whorls is represented by the overlap of the whorl. The whorl overlap is partly influenced by the whorl shape but is mainly determined by the proportion of



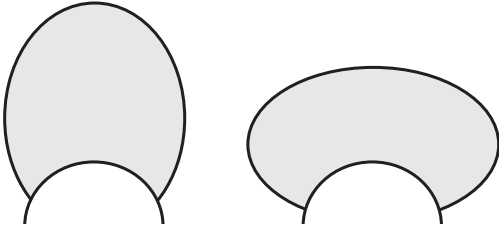


Fig. 3. Different shapes of succeeding whorl (gray area) piling up on the hemispherical preceding whorl with same  $L_i$ ,  $l$ , and  $A$  values.

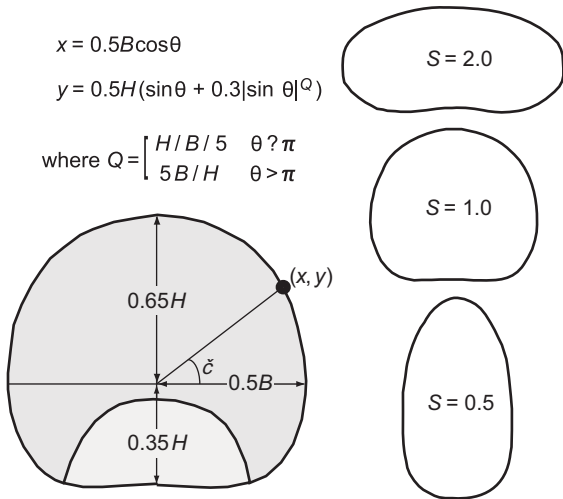


Fig. 4. Schematic diagram of an ammonoid whorl section. The shape of the whorl ( $S$ ) is represented by the closed curve illustrated here which is given by the parametric equations indicated above the diagram. Figures in the right show examples of hypothetical whorl shapes with systematically varying the  $S$  value.

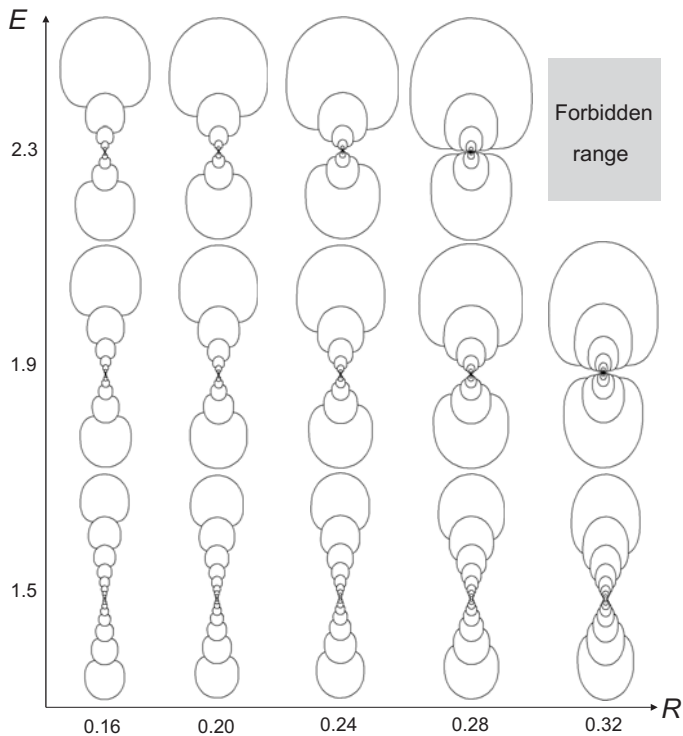


Fig. 5. Spectrum of the computer-produced ammonoids with various values of  $E$  and  $R$  when each of them is fixed throughout growth.

the circumferential length along the dorsal wall ( $l$ ) to the total perimeter of the whorl section ( $L_i$ ), which is expressed as  $R$ :

$$l = RL_i \tag{2}$$

The shape of a newly added whorl,  $S$ , is dependently determined by  $E$  and  $R$ , if the preceding whorl is given and the area of the succeeding whorl in the section,  $A$ , is specified. In the default setting of the model, the initial shell consists of a pair of semicircular whorls (Fig. 2), and isometric growth of  $A$  with respect to  $L_i$  is assumed.

$S$  depends on  $E$  and  $R$  but is not always uniquely determined by  $E$  and  $R$ . In some cases, two totally different whorl shapes are potentially generated using the same values of  $E$  and  $R$  (Fig. 3). In such cases, we chose the whorl shape which has a closer  $S$  value to the preceding whorl than the other.

In many ammonoids, the cross-sectional shape of the whorl is oval or subquadrate rather than elliptic. In computer simulations, therefore, the following parametric equations were used for producing an oval or subquadrate closed curve as a better representation of a whorl section (Fig. 4):

$$x = 0.5B\cos\theta, y = 0.5H(\sin\theta + 0.3|\sin\theta|^Q)$$

where  $Q = \begin{cases} H/B/5 & \theta \leq \pi \\ 5B/H & \theta > \pi \end{cases}$

These equations define  $x$  and  $y$  coordinates of a point on the curve at an angle  $\theta$ .  $B$  and  $H$  express the breadth and height of the whorl, respectively, and  $S$  is defined as  $B/H$ . The equations above were arbitrary designed just for approximation of the whorl shape, and there is no biological basis of its formulation. Possible varieties of whorl shape can be generated with varying the value of  $S$  (Fig. 4).

We can now define shell growth in terms of shape parameters  $E$  and  $R$ , if an isometric relationship is assumed between size parameters  $A$  and  $L_i$ , and the values of  $E$  and  $R$  are fixed throughout growth. In this model, the circumferential length along the dorsal wall of the succeeding whorl ( $l = RL_i$ ) cannot be larger than that of the external wall of the preceding whorl  $[(1-R)L_i/E]$ . Therefore, in the case of isometry, a theoretical model can be defined only within the following range of shape parameters:

$$E+1 \leq 1/R \tag{3}$$

In computer simulations, vast number of computer models with various shapes ( $S$ ) and positions of the whorl were generated, and we searched the computer-generated models for the desired combination of  $L_i$  and  $A$  which satisfies equations (1), (2), and (3) and isometric relationship between  $A$  and  $L_i$ .

## Computer simulations of shell growth

**Isometric growth.**—The effects of  $E$  and  $R$  on the shell form in cross section are shown on computer-produced shells in Fig. 5. When  $E$  and  $R$  are small, the model generates an

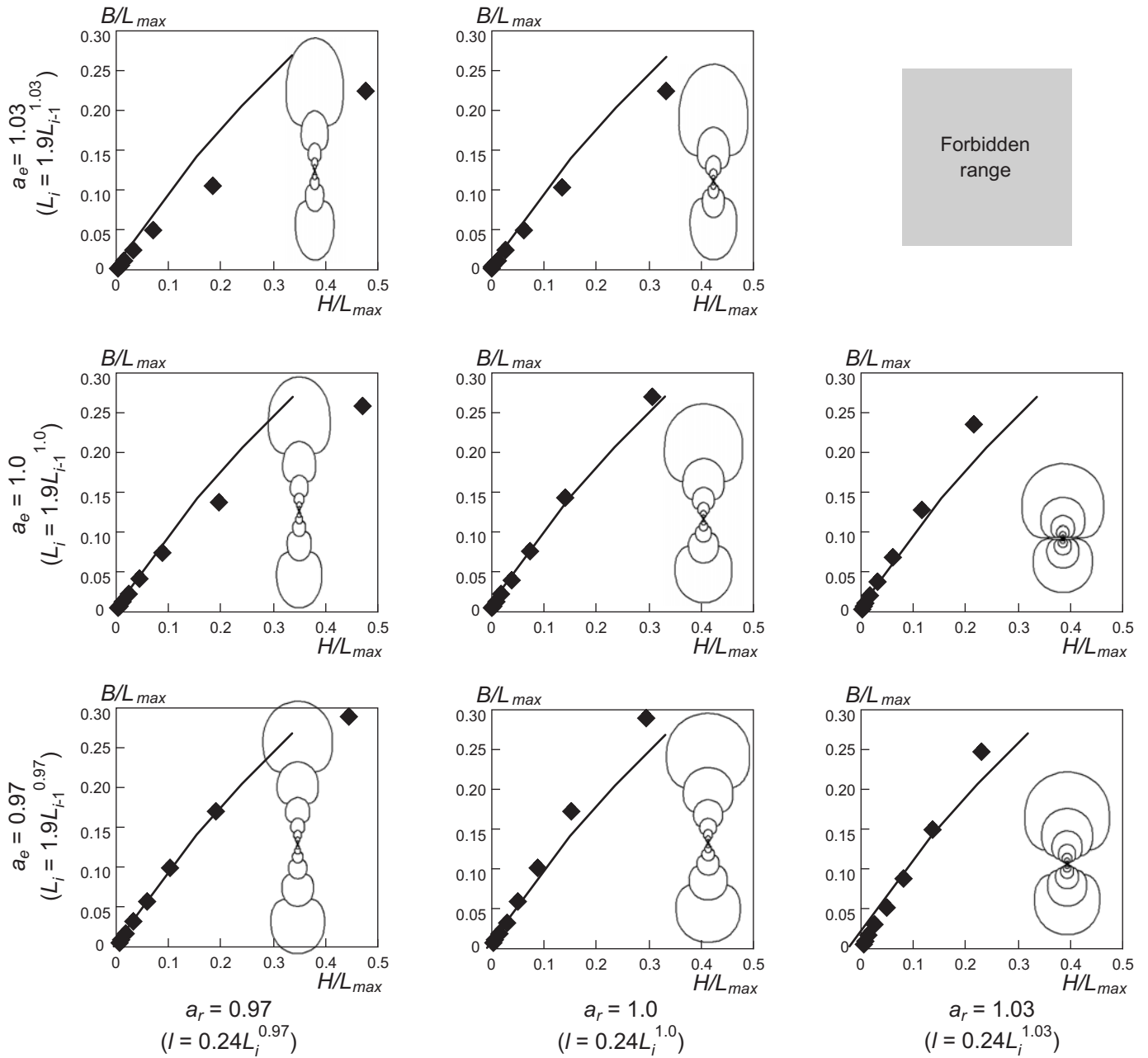


Fig. 6. Allometric growth of ammonoids. Computer models were generated with  $a_e$  and  $a_r$  values of 0.97, 1.0, and 1.3. Each diagram shows ontogenetic change in relationship between whorl height ( $H$ ) and whorl breadth ( $B$ ) that are standardized by the perimeter of the last whorl ( $L_{max}$ ). Dashed line in each diagram represents the  $B/H$  curve in the case of isometry of  $E$  and  $R$  ( $a_e = a_r = 1.0$ ). Note that  $B/H$  decreases with growth in the case of  $a_r = 0.97$ . In each model,  $e = 1.9$  and  $r = 0.24$ .

evolute conch in which the whorls overlap little so that there is a wide umbilicus. An increase in  $E$  or  $R$  tends to produce an involute shell with a narrow umbilicus. It is a matter of course that the whorl expansion rate increases with increasing  $E$ . If  $E$  and  $R$  are too large to satisfy the former condition, any theoretical form cannot be defined in the piggyback whorls model (“forbidden range” sensu Tyszka 2006).

Theoretical models based on parameter values that are fixed throughout growth do not realize ontogenetic change in shell form. In addition, models shown in Fig. 5 do not produce variations in whorl shape: all of them have subcircular

or subquadrate whorls. A greater variety can be accomplished by simulations in which  $E$  and  $R$  change with growth.

**Allometric growth.**—For representing allometric growth of ammonoid shells, the following equations were employed to define ontogenetic change of  $E$  and  $R$ :

$$L_i = eL_{i-1}^{ae}, \quad l = rL_i^{ar},$$

where  $e$ ,  $r$ ,  $a_e$  and  $a_r$ , are allometric coefficients. Parameters  $e$  and  $r$  have same effects as the enlarging ratio of the whorl perimeter and proportion of the dorsal wall respectively, and are exactly same as  $E$  and  $R$  in the case of isometry

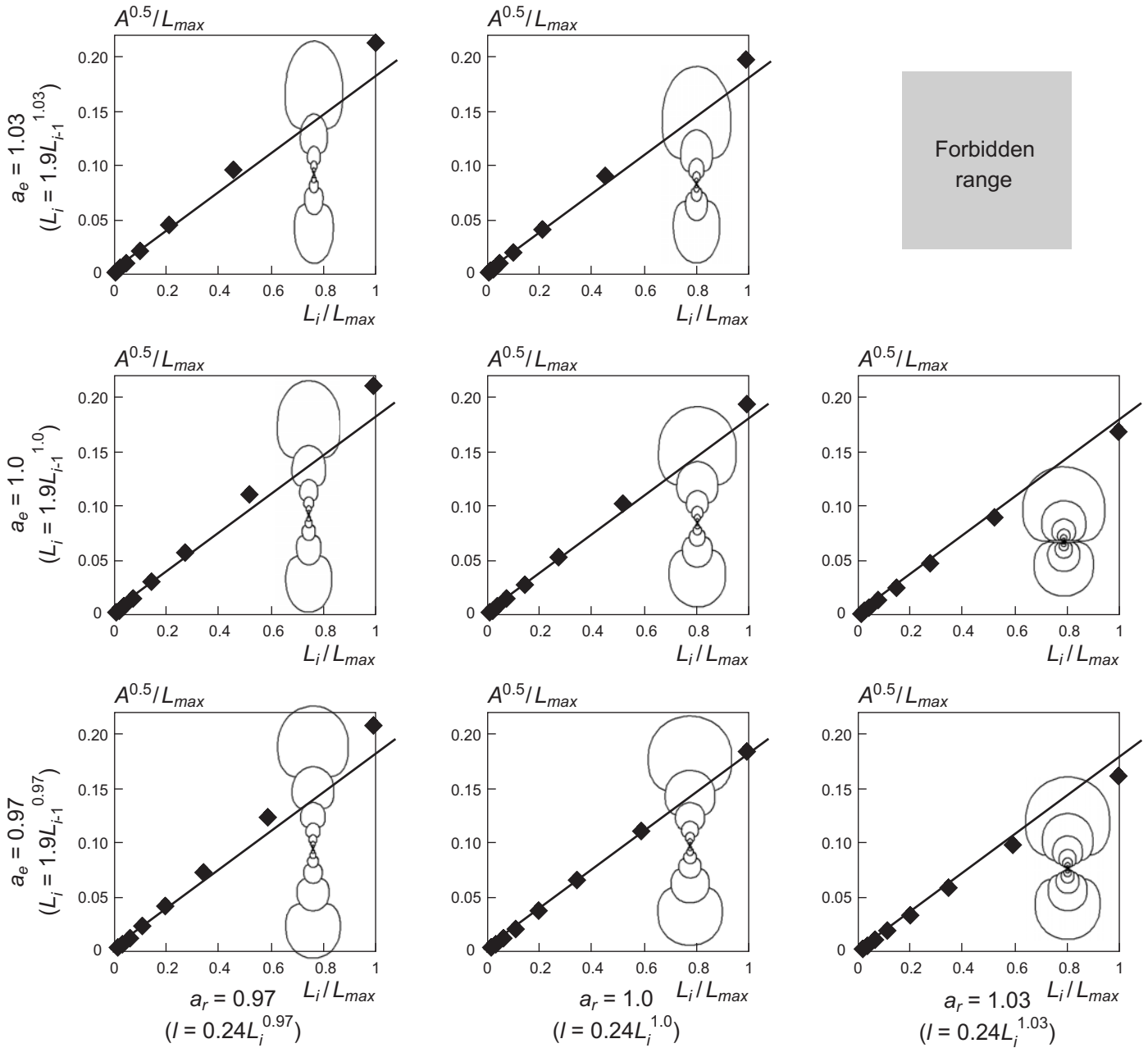


Fig. 7. Relationship between whorl perimeter ( $L_i$ ) and square root of cross-sectional area of the whorl ( $A^{0.5}$ ) which varies with  $a_e$  and  $a_r$  values. Dashed line in each diagram represents the  $A^{0.5}/L_i$  curve in the case of isometry of  $E$  and  $R$  ( $a_e = a_r = 1.0$ ).  $A^{0.5}/L_i$  is generally small when  $a_r$  has a large value. In each model,  $e = 1.9$  and  $r = 0.24$ .

( $a_e=1$  and  $a_r=1$ ). Then,  $E$  and  $R$  are given by  $eL_{i-1}^{ae-1}$  and  $rL_{i-1}^{ar-1}$ , respectively. When  $a_e>1$  or  $a_r<1$ , the portion of the whorl overlap decreases with growth. This condition inevitably causes an excessive area of the whorl section and results in positive allometry of  $A$  with respect to  $L_i$ , if  $S$  is fixed to 1.0 so as to maintain a subcircular or subquadrate whorl shape. In other words, isometric growth of  $A$  with respect to this condition requires reduction of  $S$  to produce a more compressed whorl, because a slender shape generally has a small  $A^{0.5}/L_i$  ratio. Therefore, computer simulations in the default setting produce a slender whorl and cause a decrease of  $S$  with growth when  $a_e>1$  and/or  $a_r<1$  (Fig. 6).

On the contrary, when  $a_e<1$  and/or  $a_r>1$ , the whorl overlap increases as growth proceeds, and a burly shape tends to be formed for each whorl to make up for the “deficiency” of the area (Fig. 6). However, compensation for the “deficiency” by changing the whorl shape has its limit because  $A^{0.5}/L_i$  takes a maximum value around  $S = 1$ . If  $a_e$  is small enough or  $a_r$  is large enough, the default setting, i.e., isometric growth of  $A$  with respect to  $L_i$ , can not be held. In this case, a negative allometry was allowed in the present simulations: the maximum value of  $A$  was searched within the theoretically possible range and was employed for each whorl. Consequently, when  $a_e$  is small and  $a_r$  is large,  $A^{0.5}/L_i$  tends to

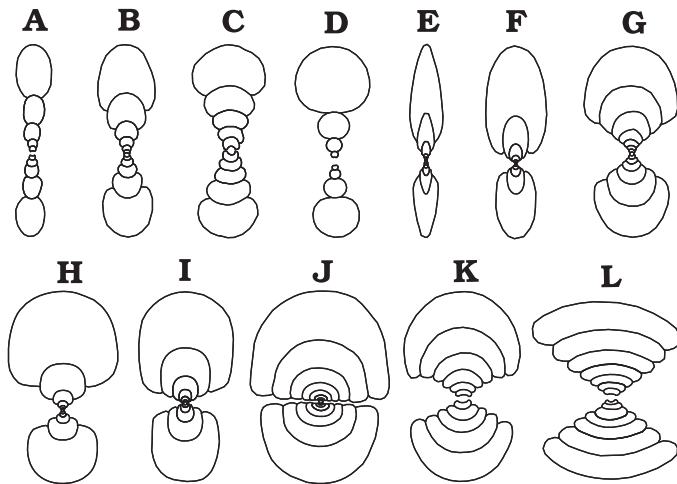


Fig. 8. Sketches of radial cross sections of ammonoids to illustrate various types of shell forms, such as planorbicone (A, B), serpenticone (C, D), oxycone (E), discocone (F), platycone (G, H), spherocone (I, J, K), and cadicone (L). A. *Paracelites elegans*. B. *Pseudoclymenia dillensis*. C. *Tropigastrites lahotanus*. D. *Pterolytoceras* sp. E. *Beloceras* sp. F. *Phylloceras consanguineum* Gemmellaro. G. *Craspedites* sp. H. *Tetragonites glabrus*. I. *Damesites sugata*. J. *Goniatites multiliratus* Gordon. K. *Latanarcestes* sp. L. *Cabrieroceras* sp.

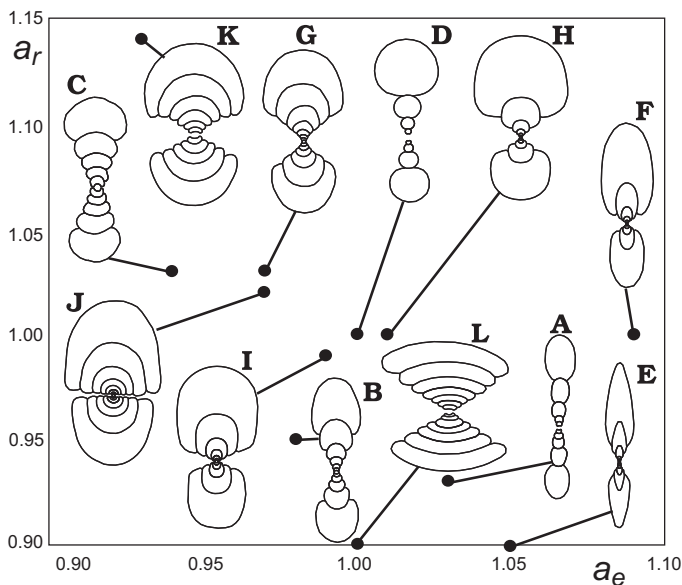


Fig. 9. Theoretical morphospace composed of  $a_e$  and  $a_r$  displaying several examples of computer-generated ammonoids that represent observed types illustrated in Fig. 8. A.  $e = 1.5$  and  $r = 0.27$ . B.  $e = 1.75$  and  $r = 0.34$ . C.  $e = 2.0$  and  $r = 0.17$ . D.  $e = 2.3$  and  $r = 0.05$ . E.  $e = 2.0$  and  $r = 0.6$ . F.  $e = 1.7$  and  $r = 0.27$ . G.  $e = 1.8$  and  $r = 0.24$ . H.  $e = 2.3$  and  $r = 0.21$ . I.  $e = 2.3$  and  $r = 0.32$ . J.  $e = 1.95$  and  $r = 0.28$ . K.  $e = 1.8$  and  $r = 0.10$ . L.  $e = 1.6$  and  $r = 0.8$ .

decrease with increasing size so that it has a smaller value in comparison with other cases (Fig. 7). If  $a_e$  and  $a_r$  are both considerably larger than zero, any theoretical form cannot be defined.

If appropriate values of  $a_e$  and  $a_r$  are used with various combinations of  $e$  and  $r$ , many varieties of shell forms observed in actual ammonoids as shown in Fig. 8, can be pro-

duced (Fig. 9). An evolute conch with a compressed whorl, or planorbicone, is generated when  $a_r$  is small and  $a_e$  is around 1.0 (Fig. 9A, B). A model with a large value of  $a_e$  can produce a discoidal form with a compressed whorl and a narrow umbilicus (Fig. 9F). A spheroconic shell which is characterized by a subglobular whorl and involute coiling is readily reproduced when  $a_r$  is around one and  $a_e$  is slightly smaller than 1.0 (Fig. 9G, I, J). If  $r$  is large and  $a_r$  is small, a theoretical model possessing a large value of  $e$  tends to generate an oxyconic form with an extremely compressed whorl (Fig. 9E), while a model with a small  $e$  value can produce a cadiconic form with a depressed whorl (Fig. 9L). Other observed forms, such as are seen in Fig. 1C, D, H, K, can be generated by our model (Fig. 9C, D, H, K).

## Morphospace analysis

**Computer simulations.**—The piggyback whorls model assumes that the shape of a whorl section depends on the balance between the perimeter and area of the whorl section. This assumption may cause biased occupation patterns of theoretically possible forms in the morphospace which defines the relationship between growth parameters and cross-sectional form. If so, analysis of the distribution of actual forms in relation to the theoretical morphospace allows us to test the assumption. Since allometric growth is defined by  $e$ ,  $r$ ,  $a_e$  and  $a_r$ , the morphospace composed of these growth parameters and a dependent subsidiary parameter such as  $S$  should be high-dimensional. Since accurate estimations of  $e$  and  $r$  from specimens are not easy, the relationship between pairs of  $a_e$  and  $a_r$  and subsidiary parameters including Raup's ones was analyzed to assess explicability of the piggyback whorls model for well-known correlations among Raup's parameters.

In addition to  $S$ , the following three subsidiary parameters were also assessed: whorl expansion rate,  $W_c$ ; width of umbilicus, or Raup's  $D$  (Fig. 10); and ratio of the square root of the area to the perimeter,  $A^{0.5}/L_i$ . Raup (1967) defined the whorl expansion rate as the rate of increase in the diameter of an ammonoid shell per revolution: it can be defined independent of  $S$  in the case of isometry. However, increase in compression of the whorl during growth results in increase in the whorl expansion rate defined by Raup (1967). In this study, for avoidance of algebraic dependency,  $W_c$  was defined as the ratio of the distance from the coiling axis to the center of a given whorl to that of the preceding one (Fig. 10). The center of the whorl is defined as the point on the median line equally distant from the ventral and dorsal extremes of the whorl. When  $a_e < 1$  or  $a_r < 1$ , the values of  $S$ ,  $W_c$ ,  $D$ , and  $A^{0.5}/L_i$  generally change with growth. In the present analysis, these subsidiary parameters were estimated for the latest whorl both in theoretical models and actual specimens.

Computer simulations were performed to establish theoretical morphospaces composed of  $a_e$ ,  $a_r$ , and  $S$ ,  $W_c$ ,  $D$  or



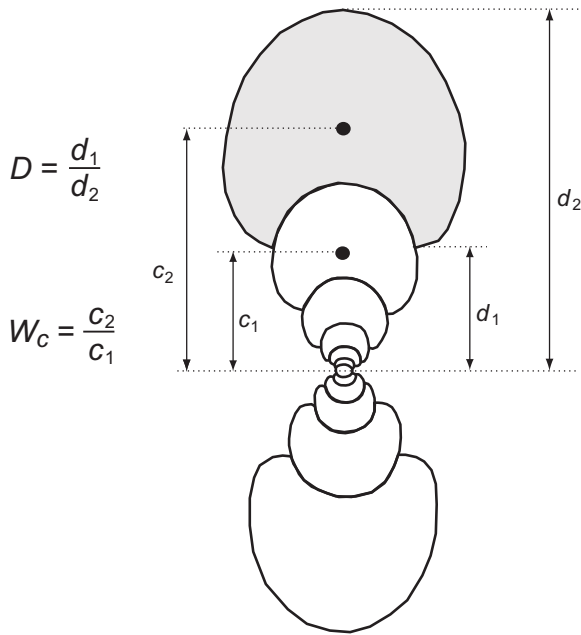


Fig. 10. Measurements of the whorl expansion rate ( $W_c$ ) and width of umbilicus ( $D$ ). Note that  $c_1$  and  $c_2$  are defined as the distances from the coiling axis to the centers of the whorls.  $d_1$  and  $d_2$  are distances of umbilical seams from the coiling axis.

$A^{0.5}/L_i$  with various values of  $e$ ,  $r$ ,  $a_e$ , and  $a_r$ . A total of 200 theoretical models were simulated. In each model, values of  $e$ ,  $r$ ,  $a_e$  and  $a_r$  were randomly chosen from the normally distributed populations with the following means and standard deviations:  $\mu = 2$ ,  $\sigma = 0.2$  for  $e$ ;  $\mu = 0.25$ ,  $\sigma = 0.05$  for  $r$ ;  $\mu = 1$ ,  $\sigma = 0.05$  for  $a_e$  and  $a_r$ . The subsidiary parameters associated with each combination of  $a_e$  and  $a_r$  attain various values depending on  $e$  and  $r$ , but are obviously correlated with  $a_e$  and/or  $a_r$  (Fig. 11A–D).  $S$  tends to decrease as  $a_r$  decreases or  $a_e$  increases (Fig. 11A). In this case,  $a_r$  is the predominant factor in determination of the shape parameters. On the other hand,  $A^{0.5}/L_i$  generally increases with decreasing  $a_r$  (Fig. 11B). As mentioned above, a small  $a_r$  or large  $a_e$  value reduces the portion of the whorl overlap with growth and potentially yields an excessive area of the whorl section to increase  $A^{0.5}/L_i$  (Fig. 11B). In this condition, balancing between  $A$  and  $L_i$  reduces the value of  $S$  to produce a compressed whorl (Fig. 11A).  $W_c$  dominantly depends on  $a_e$  (Fig. 11C), and this relation is rather obvious: a large  $a_e$  value readily causes a large whorl expansion rate.  $D$  has a tendency to decrease with increase either of  $a_e$  or  $a_r$ , but is mainly influenced by  $a_r$  (Fig. 11D). As a natural consequence of comparison of morphospaces between Fig. 11A and Fig. 11C, a clear negative correlation between  $S$  and  $W_c$  was generated in our computer simulations (Fig. 11E). For a similar reason, there is a positive correlation between  $S$  and  $D$  (Fig. 11F), and consequently a negative one between  $S$  and  $W_c$ , which have been reported in many ammonoids (Bayer and McGhee 1984; Saunders and Swan 1984; Swan and Saunders 1987; Dommergues et al. 1996; Saunders and Work 1996). These

correlations derived from the assumption of the piggyback whorls model: balancing between  $A$  and  $L_i$  determines  $S$ .

**Biometric analyses.**—The occupation patterns of hypothetical models in the morphospaces shown in Fig. 11 were compared with the distribution of actual ammonoids to test our assumption. To assess the pattern of morphological diversity of actual ammonoids,  $a_e$ ,  $a_r$ ,  $S$ ,  $W_c$ ,  $D$  and  $A^{0.5}/L_i$  were estimated on 123 specimens of 115 species, belonging to 72 families (see Appendix 1). Since each species was represented by one or a few specimen, the present analyses and following discussions basically dealt with shape variation above species level. Our material covers six ammonoid orders spanning from the Devonian to the Cretaceous.

Each specimen was first cut using an electric diamond saw vertical to the median plane. The half section was polished with graded series of silicon-carbide powders along the radial plane. A sheet of cellophane tape was pressed onto the polished surface to make the structure on the surface clearly visible. The polished surface was laid onto the image scanner (Epson GT-F600), and an image of the surface was captured and saved as a computer bitmap file. Two-dimensional coordinate data along the periphery of the whorl were collected on the bitmap image using a digitizing program written in Visual Basic 6.0 (Ubukata 2004, 2005), and then, the perimeter along the sectioned whorl was calculated. We measured to the outside of the shell wall because morphogenesis of the external shell surface is not affected by shell thickening. The height, breadth and center of the whorl were determined by the coordinate data to obtain  $S$ ,  $D$  and  $W$ . For measuring  $A$ , the periphery of each whorl was traced and colored differently on a personal computer using Justsystem Hanako PhotoRetouch 2004 program, and its area was measured by counting pixels (Ubukata 2001). For this counting, a program written in Visual Basic 6.0 was used. Consequently,  $a_e$  and  $a_r$  were obtained as slopes of reduced major axes of  $\log L_i$  against  $\log L_{i-1}$ , and  $\log l$  against  $\log L_i$ , respectively (Jones 1937). For details of the regression method using the reduced major axis, refer also to Imbrie (1956). Multiple regression analyses were conducted for parameters on  $a_e$  and  $a_r$ .

The range of forms of actual ammonoids examined was consistent with the occupation patterns of theoretical models in morphospace shown in Fig. 11. In actual ammonoids,  $S$  appears to decrease with increase of  $a_e$  or decrease of  $a_r$  in Fig. 12A. A multiple regression analysis revealed a significant negative partial correlation between  $a_e$  and  $S$ , and a positive correlation between  $a_r$  and  $S$ , and the latter is especially prominent (Table 1). Fig. 11B indicates a negative relationship between  $A^{0.5}/L_i$  and  $a_r$ , and a multiple regression analysis provided a significant trend to decreasing  $A^{0.5}/L_i$  as a function of  $a_r$  (Table 1). The value of  $W_c$  seems to increase as  $a_e$  increases (Fig. 12C), which was supported by multiple regression analysis (Table 1). A significant negative partial correlation was found between  $a_r$  and  $D$  (Table 1, Fig. 12D). Biometric analyses also revealed a clear negative or inverse relationship between  $S$  and  $W_c$  (Fig. 12E) and a weak positive correlation be-



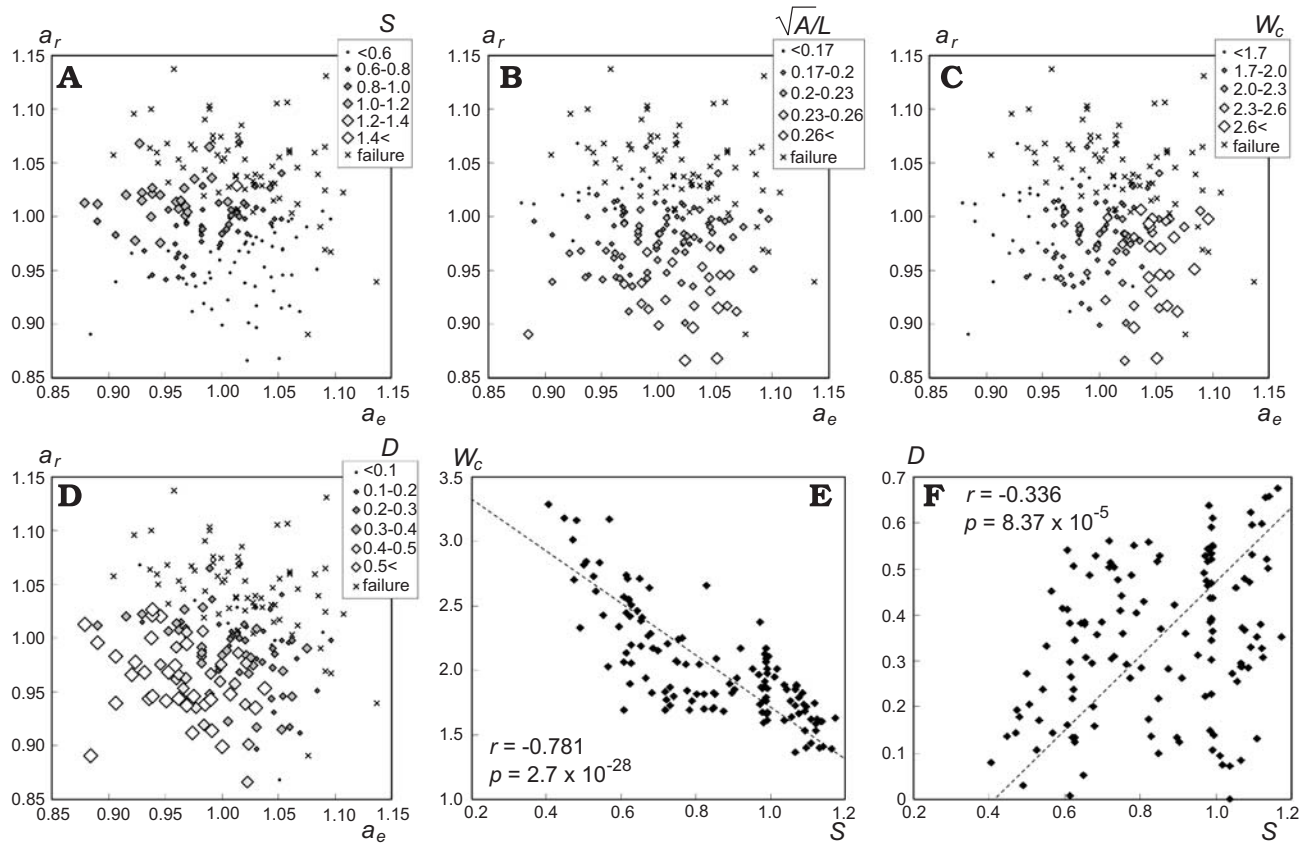


Fig. 11. Results of computer simulations. A–D, theoretical morphospaces based on the piggyback whorls model composed of  $a_e$  and  $a_r$ , showing values of shape parameters ( $S$ ,  $A^{0.5}/L_i$ ,  $W_c$ ,  $D$ ) of a theoretical model corresponding to each combination of  $a_e$  and  $a_r$ . E, negative correlation between  $W_c$  and  $S$  values obtained from theoretical models. F, positive correlation between  $D$  and  $S$ . The values of  $S$ ,  $A^{0.5}/L_i$ ,  $W_c$  or  $D$  are exhibited by the size of the plots, and  $\times$  indicates that a “forbidden” combinations of  $a_e$ ,  $a_r$ ,  $e$ , and  $r$  was employed in each simulation resulting in failure of defining form (A–D).

Table 1. Results of multiple regression analyses.

Regression of		Standard partial regression coefficient	Partial correlation coefficient	$p$
$S$	$ae$	-0.3319	-0.3362	0.0002
	$ar$	0.2220	0.2323	0.0100
$W$	$ae$	0.2860	0.2887	0.0013
	$ar$	-0.2237	-0.2295	0.0110
$A^{0.5}/L$	$ae$	0.0509	0.0650	0.4769
	$ar$	-0.6430	-0.6352	0.0000
$D$	$ae$	-0.0849	-0.0891	0.3294
	$ar$	-0.4209	-0.4053	0.0000

tween  $S$  and  $D$  (Fig. 12F). These results are generally concordant with results of computer simulations (Fig. 11), and thus support the assumption of the present model.

## Discussion

**Correlations among Raup’s parameters.**—Saunders and Swan (1984) and Swan and Saunders (1987) analyzed Raup’s  $W$ - $D$ - $S$  morphospace in various Paleozoic ammonoids and identified correlations among these parameters that are not ex-

plicable in terms of hydrodynamics or hydrostatics: a negative correlation between  $W$  and  $D$ , and a positive one between  $S$  and  $D$ , that have been confirmed by several authors (Bayer and McGhee 1984; Dommergues et al. 1996; Saunders and Work 1996), and were also reconfirmed in the present study. Swan and Saunders (1987) suggested that these correlations originated in restrictions imposed by the relative aperture height and aperture area with respect to shell size. They found that the major axis of plots was distributed along aperture-height curves in the  $W$ - $D$  morphospace and  $S$ - $D$  plots run parallel to aperture-area curves (Swan and Saunders 1987: figs. 8, 9). Swan and Saunders (1987) supposed preferences for aperture height and aperture area that make a moderately deep U-shaped cross section to avoid biomechanical disadvantages.

In the present piggyback whorls model, isometric relationship is assumed between the area of the whorl cross section and the whorl perimeter. It may be safe to say that this assumption of the model is concordant with the Swan and Saunders (1987) hypothesis of restrictions on aperture height and aperture area. The piggyback whorls model displayed how constructional bias imposed by the area of whorl section work on the occupation pattern of morphospace: changing the geometry of shell coiling inevitably results in change in the aperture shape so as to keep the constant relative area of

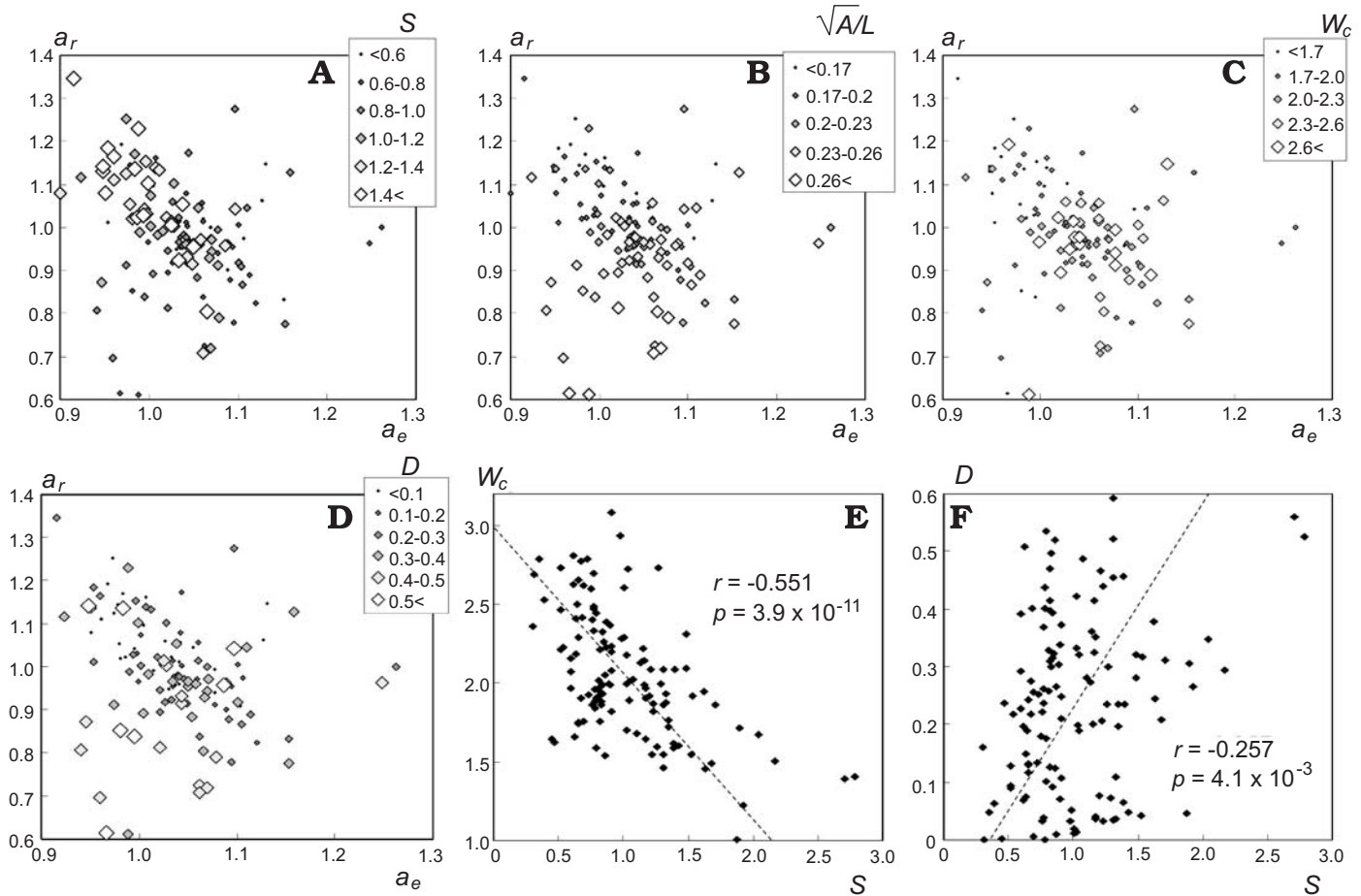


Fig. 12. The same morphospaces as Fig. 11 showing the distribution of 115 species based on measurements.

the aperture with respect to its perimeter. Since the values of Raup's parameters depend on  $r$  and  $a_r$ , correlations among Raup's parameters can be regarded as axiomatic consequences of algebraic interdependence among them, if an isometric relationship is assumed between the area and perimeter of the whorl cross section. Concerning the Raup's model, Schindel (1990) pointed out that, the dorsal shell margin of two whorls can move away from the coiling axis at the same rate but with different  $D$  values, because of an algebraic interdependence between  $W$  and  $D$  under a fixed displacement rate of the dorsal shell margin (see also Stone 1996). Origination of a correlation between parameters from an assumed condition (e.g., fixed displacement rate, or isometry between area and perimeter) can be attributed to a simple mathematical problem; however, the assumed condition should be placed in a biological context. In the case of the present study, correlations among Raup's parameters may be interpreted as "constructional linkages" associated with particular configurations of aperture height and aperture area that were assumed by Swan and Saunders (1987).

**Buckman's Law of Covariation.**—The piggyback whorls model may partly explain Buckman's Law of Covariation between ornament and shell shape: strongly ornamented am-

monoids tend to have an evolute conch and circular whorls (Westermann 1966). This "law" has been confirmed by the morphologic analysis of large population samples of some Triassic ceratites (Dagys and Weitschat 1993; Dagys et al. 1999; Dagys, 2001) and a Cretaceous ammonite *Neogastropilites* (Kennedy and Cobban, 1976), and this phenomenon has been explained in terms of a reaction diffusion model (Guex et al. 2003) and a simple principle of proportionality (Hammer and Bucher 2005a).

If an ornamented closed curve is *a priori* given for whorl cross section as an initial setting, same computer simulations as shown in Fig. 11D tend to generate more depressed whorls with larger  $S$  values in the piggyback whorls model (Fig. 13), where compression is *a posteriori* determined. In terms of the piggyback whorls model, formation of heavy ornaments makes a long periphery of the whorl and reduces the  $A^{0.5}/L_i$  ratio. In this case, employing a circular aperture maximizes the area of the whorl section and then balances between  $A$  and  $L_i$ , as in the case that a burly aperture makes up for the deficiency of the area caused by increase of the whorl overlap with growth (Fig. 6). Since shell involution is practically determined by  $r$  and  $a_r$  that are *a priori* given in the present simulations, the relationship between evolute conch and ornamentation was not reproduced.

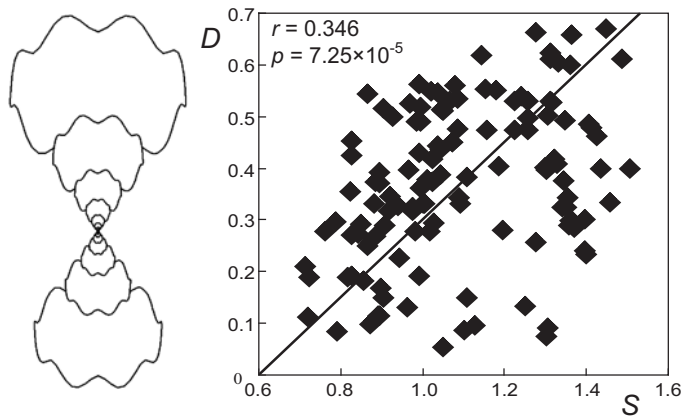


Fig. 13. Results of same computer simulations as shown in Fig. 11 with an ornated whorl shape. In order to produce an ornated closed curve, the following parametric equations were used:  $x = 0.5B\cos\theta$ ,  $y = 0.5H(\sin\theta + 0.3\sin 4\theta/Q)$ , where  $Q = H/B/5$  if  $\theta \leq \pi$ , while  $Q = 5B/H$  if  $\theta > \pi$ . Note that simulated  $S$  values tend to be large as compared with Fig. 11F.

Yacobucci (2004) found that Cretaceous acanthoceratid ammonites with highly variable shell shapes did not show variable patterns of ornamentation, although the Buckman's Law should expect a positive correlation of variations between shell shape and rib features. She suggested that ornament morphogenesis is genetically and/or developmentally more controlled than is shell shape. If gross shell geometry is more variable than shell sculpture, heavy ornamentation which may lead changes in whorl shape. However, this explanation based on the present model can only be applied to tubercles and spiral ribs, because radial ribbing involves little change in aperture shape, which in turn does not influence  $A^{0.5}/L_t$ .

**Hydrostatics and hydrodynamics.**—As a result of computer simulations based on the piggyback whorls model, we expected that positive allometry of whorl periphery causes the whorl to become more slender and more rapidly expanding (Fig. 11A, C). Morphometric analysis also found that the allometric coefficient of periphery is negatively correlated with whorl compression but is positively correlated with whorl expansion rate (Fig. 12A, C, Table 1). Positive allometry of the whorl periphery means acceleration of the lateral expansion of the soft body. If this is not attended with acceleration of increase in total soft-body mass, body-chamber length is necessarily shortened to compensate rapid expansion of the diameter. Therefore, the process of building a compressed whorl with a high expansion rate may result in formation of a short body chamber. This study did not examine the body-chamber length because most of the specimens utilized did not preserve the entire body chamber. However, a negative correlation between body-chamber length and whorl expansion rate has been observed in some taxa (Okamoto 1996; Klug and Korn 2004). From the view point of hydrostatics, this correlation is theoretically expected if neutral buoyancy is presumed (Saunders and Shapiro 1986; Ebel 1990). The piggyback whorls model may explain how ammonoids constructed neutrally buoyant shell morphology;

constraints imposed by the relative area of the aperture with respect to its perimeter may cause a negative correlation between body-chamber length and whorl expansion rate.

In addition, a negative correlation between  $S$  and whorl expansion rate (Fig. 12E) strongly suggests a negative correlation between whorl compression and body chamber length. A shell with a short body chamber has high hydrostatic stability which allows the animal's fast locomotion without rotation (Trueman 1941; Raup 1967; Saunders and Shapiro 1986; Ebel 1990; Okamoto 1996). In terms of hydrodynamics, a compressed form generally has a lower drag coefficient than robust one. However, a compressed shell has a smaller amount of retractor muscle and a larger surface area which generates a larger frictional drag than a thicker form. Thus, the compressed form is effective for reducing drag only at a high Reynolds number which represents a large size and/or a fast swimming velocity (Jacobs 1992; Jacobs and Chamberlain 1996; Seki et al. 2000). Positive allometry of whorl periphery readily realizes an ideal coupling of morphological characters for a fast-swimming ammonite, because of the constructional linkage between whorl compression and whorl expansion rate. Furthermore, it also results in an ontogenetic change from a less compressed to more compressed shape: such ontogenetic trend is suitable for swimming because smaller forms that swim slowly benefit from less compressed shapes (Jacobs 1992; Jacobs and Chamberlain 1996).

In general terms, this study suggests that constructional biases as assumed in the piggyback whorls model can provide a functional coupling of morphological characters. Functional and developmental integrations of characters have been focused in the context of developmental constraints (Alberch 1980, 1982; Kauffman 1983; Schwenk and Wagner 2001; Richardson and Chipman 2003), and have been discussed from viewpoints of pleiotropy (Cheverud 1996; Raff 1996; Adams 1998; Moore et al. 2004; Klingenberg 2005) or epigenetic processes (Müller 1989; Morita 1993). Although this study did not attempt to elucidate developmental mechanism of morphogenesis of functionally coupling characters, the piggyback whorls model expects "constructional linkages" between aperture shape and coiling geometry.

## Concluding remarks and prospects

The piggyback whorls model introduced for mimicking ammonoid shell growth is based on the geometric balance among perimeters of the external and internal portions and area of the whorl section. The balance determines degree of compression and involution of whorls. None of the growth parameters used in this model directly defines the shape of the aperture and geometry of shell coiling. Theoretical forms generated by using the piggyback whorls model has a potential to explain well-known correlations among whorl expansion rate, width of umbilicus and whorl shape (Fig. 11E, F), as consequences from constructional biases imposed by isometric relationship between perimeter and area of the whorl cross section.



The piggyback whorls model is not designed to simulate openly coiled or uncoiled form, because it attributes great variation in aspect ratio of whorl section to ontogenetic change of whorl overlap and whorl expansion rate. The present model is not comprehensive enough to reproduce various teratological patterns observed in actual ammonoids. In addition, it is not sufficient for analysis of the three-dimensional properties of shell form. The model does not address the effect of pattern of relative growth rate around the aperture on the aperture shape. Combining the present model with the aperture map model (Rice 1998) or aperture growth vector model (Hammer and Bucher 2005b) may be required to show how three-dimensional shell form is determined.

Nevertheless, the present model is useful for analysis of ontogenetic variation of ammonoids, which is usually based on cross sections of shells. There is a prospect that the model may explain how to realize a functional coupling between hydrostatic and hydrodynamic characters and how to produce the Buckman's Law of Covariation between rib features and shell shapes. The approach introduced here is also available for studies on nautilids, and might be applicable to gastropod shell morphology after some necessary modifications.

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## Appendix 1

Specimens examined and measured values of parameters. All specimens have the prefix UMUT. The higher taxonomy follows Korn and Klug (2002) for Devonian ammonoids, Bogoslovskaya et al. (1999) for Carboniferous and Permian ammonoids, and Page (1996) for Mesozoic ammonoids.

Order	Family	Species	$a_e$	$a_r$	S	$W_c$	D	$A^{0.5}/L$	Specimens	Age	Locality	
Agoniatitida	Latanarcestidae	<i>Latanarcestes</i> sp.	1.01	1.14	1.22	1.55	0.21	0.17	PM-29049	Devonian	Taouz, Morocco	
	Agoniatitidae	<i>Fidelites</i> sp.	1.06	1.06	0.75	2.60	0.18	0.24	PM-29050	Devonian	Erfoud, Morocco	
		<i>Achguigites</i> sp.	1.12	0.83	0.66	2.29	0.12	0.21	PM-29051	Devonian	Erfoud, Morocco	
	Ponticeratidae	<i>Pseudoproboceras costulatum</i>	1.09	0.96	0.82	1.98	0.31	0.23	PM-29052	Devonian	Taouz, Morocco	
	Beloceratidae	<i>Beloceras</i> sp.	1.11	0.97	0.30	2.36	0.16	0.16	PM-29053	Devonian	Erfoud, Morocco	
	Anarcestidae	<i>Anarcestes mateviai</i>	1.04	0.98	0.91	3.08	0.21	0.23	PM-29054	Devonian	Erfoud, Morocco	
	Werneroceratidae	<i>Praewerneroceras hollardi</i>	1.00	1.10	1.62	1.94	0.38	0.21	PM-29055	Devonian	Taouz, Morocco	
	Cabrieroceratidae	<i>Cabrieroceras</i> sp.	0.95	1.14	2.79	1.40	0.53	0.19	PM-29056	Devonian	Taouz, Morocco	
	Sobolewiidae	<i>Subanarcestes</i> sp.	0.99	1.03	1.63	1.46	0.25	0.19	PM-29057	Devonian	Erfoud, Morocco	
	Pharciceratidae	<i>Stenopharciceras viseireuge</i>	1.04	1.17	0.82	1.95	0.13	0.19	PM-29058	Devonian	Taouz, Morocco	
Goniatitida	Tornoceratidae	<i>Tornoceras</i> sp.	1.03	1.01	0.77	1.86	0.03	0.17	PM-29059	Devonian	Fezzou, Morocco	
		<i>Epitornoceras mithracoides</i>	0.98	1.14	0.69	1.76	0.00	0.16	PM-29060	Devonian	Taouz, Morocco	
	Pseudoclymeniidae	<i>Pseudoclymenia dillensis</i>	1.04	0.91	0.79	1.84	0.40	0.22	PM-29061	Devonian	Aktyubinsk, Kazakhstan	
	Cheiloceratidae	<i>Cheiloceras unclulosum</i>	0.96	1.11	1.23	1.82	0.03	0.18	PM-29062	Devonian	Taouz, Morocco	
	Sporadoceratidae	<i>Sporadoceras</i> sp. 1	0.98	1.17	1.03	1.70	0.01	0.16	PM-29063	Devonian	Taouz, Morocco	
		<i>Sporadoceras</i> sp. 2	1.06	1.05	1.00	2.29	0.02	0.13	PM-29064	Devonian	Taouz, Morocco	
		<i>Sporadoceras muensteri</i>	1.01	1.16	0.78	1.89	0.00	0.15	PM-29065	Devonian	Aktyubinsk, Kazakhstan	
	Prolobitidae	<i>Prolobites delphinus</i>	0.96	1.16	1.68	1.49	0.21	0.17	PM-29066	Devonian	Aktyubinsk, Kazakhstan	
	Pseudohaloritidae	<i>Neoaganides</i> sp.	1.10	0.91	0.98	2.28	0.05	0.20	PM-29067	Carboniferous	Texas	
	Girtyoceratidae	<i>Girtyoceras meslerianum</i>	1.10	1.01	0.79	2.44	0.18	0.19	PM-29068	Carboniferous	Jackforth Creek, Oklahoma	
	Goniatitidae	<i>Goniatites</i> aff. <i>crenestrina</i>	0.99	1.02	1.42	1.60	0.05	0.18	PM-29069	Carboniferous	Jackforth Creek, Oklahoma	
		<i>Goniatites multiliratus</i>		1.02	0.99	1.18	1.90	0.04	0.18	PM-29070	Carboniferous	Jackforth Creek, Oklahoma
				0.98	1.02	1.30	2.08	0.04	0.19	PM-29071		
	Glaphyritidae	<i>Glaphyrites hyattianus</i>	0.99	1.23	1.54	1.92	0.32	0.21	PM-29072	Carboniferous	Oklahoma	
		<i>Glaphyrites clinei</i>		1.05	0.95	1.89	1.72	0.31	0.21	PM-29073	Carboniferous	Oklahoma
				1.03	1.01	1.71	1.86	0.31	0.21	PM-29074		
	Reticuloceratidae	<i>Retites semiretia</i>	1.04	1.05	2.04	1.67	0.35	0.20	PM-29075	Carboniferous	Thompson Co., Arkansas	
	Bisatoceratidae	<i>Bisatoceras</i> sp.		1.00	1.03	1.35	1.72	0.20	0.20	PM-29076	Carboniferous	Oklahoma
				1.06	0.97	1.33	1.87	0.11	0.20	PM-29077		
	Thalassoceratidae	<i>Thalassoceras gemmellaroi</i>	1.03	1.06	0.87	2.39	0.01	0.19	PM-29078	Permian	Actasty R., S. Ural, Kazakhstan	
	Adrianitidae	<i>Crimites subkrotovi</i>	0.95	1.08	1.52	1.54	0.04	0.19	PM-29079	Permian	Actasty R., S. Ural, Kazakhstan	
	Vidrioceratidae	<i>Peritrochia typicus</i>		0.99	1.04	1.20	1.92	0.08	0.19	PM-29080	Permian	Actasty R., S. Ural, Kazakhstan
				0.90	1.08	1.29	1.99	0.07	0.19	PM-29081		
		<i>Peritrochia invaribilis</i>	0.98	1.05	1.39	1.62	0.06	0.17	PM-29082	Permian	Actasty R., S. Ural, Kazakhstan	
	Neostacheoceratidae	<i>Stacheoceras undatus</i>	1.09	0.88	0.77	2.48	0.24	0.23	PM-29083	Permian	Mangyskhav, Russia	
	Paragastrioceratidae	<i>Paragastrioceras</i> sp.	1.06	0.96	0.85	2.26	0.32	0.24	PM-29084	Permian	Actasty R., S. Ural, Kazakhstan	
		<i>Uraloceras involutum</i>	1.02	1.01	1.39	1.59	0.46	0.23	PM-29085	Permian	Actasty R., S. Ural, Kazakhstan	
Metalegoceratidae	<i>Metalegoceras</i> sp.	0.98	1.14	2.71	1.39	0.56	0.20	PM-29086	Permian	Actasty R., S. Ural, Kazakhstan		
Eothinitidae	<i>Eothinites kargalensis</i>	1.10	1.04	1.31	1.55	0.59	0.24	PM-29087	Permian	Actasty R., S. Ural, Kazakhstan		
Popanoceratidae	<i>Popanoceras annae</i>	1.04	1.08	0.79	1.91	0.10	0.17	PM-29088	Permian	Actasty R., S. Ural, Kazakhstan		
Clymeniida	Cymaclymeniidae	<i>Cymaclymenia</i> sp. 1	1.06	1.01	0.76	2.40	0.22	0.21	PM-29089	Devonian	Morocco	
		<i>Cymaclymenia</i> sp. 2	1.03	0.92	0.74	1.92	0.25	0.24	PM-29090	Devonian	Morocco	
	Platyclymeniidae	<i>Platyclymenia</i> sp. 1	1.25	0.96	0.78	1.96	0.44	0.26	PM-29091	Devonian	Morocco	
		<i>Platyclymenia</i> sp. 2	1.16	1.13	0.82	1.76	0.40	0.25	PM-29092	Devonian	Morocco	
		<i>Platyclymenia pompeckyi</i>	1.03	0.96	0.90	1.82	0.37	0.24	PM-29093	Devonian	Aktyubinsk, Kazakhstan	
	Clymeniidae	<i>Oxyclymenia</i> sp.	0.98	0.85	0.80	1.59	0.54	0.26	PM-29094	Devonian	Morocco	
Wocklumeriidae	<i>Wocklumeria sphaeroides</i>	0.95	1.18	1.93	1.23	0.26	0.17	PM-29095	Devonian	Wocklum, Germany		

Order	Family	Species	$a_e$	$a_r$	S	$W_c$	D	$A^{0.5}/L$	Specimens	Age	Locality
Prolecanitida	Daraelitidae	<i>Boesites</i> sp.	1.11	1.04	0.89	1.99	0.30	0.25	PM-29096	Carboniferous	Rochelle, Texas
		<i>Daraelites elegans</i>	1.11	0.89	0.68	2.77	0.22	0.23	PM-29097	Permian	Actasty R., S. Ural, Kazakhstan
	Propinacoceratidae	<i>Akmilleria electraensis</i>	1.13	1.06	0.39	2.52	0.06	0.17	PM-29098	Permian	White Pine Country, Nevada
	Medliocottiidae	<i>Medlicottia intermedia</i>	1.13	1.15	0.35	2.79	0.05	0.17	PM-29099	Permian	Actasty R., S. Ural, Kazakhstan
Pronoritidae	<i>Neopronorites skvorzovi</i>	1.03	1.01	0.72	2.79	0.13	0.21	PM-29100	Permian	Actasty R., S. Ural, Kazakhstan	
Ceratitida	Xenodiscidae	<i>Xenocelites youngi</i>	1.05	0.97	0.60	1.97	0.29	0.22	MM-29102	Triassic	Crittenden Spring, Nevada
		<i>Xenocelites subevolutus</i>	0.94	0.81	0.83	1.88	0.50	0.26	MM-29103	Triassic	Spitsbergen, Norway
	Paraceltitidae	<i>Paracelites elegans</i>	0.97	0.61	0.62	1.66	0.51	0.26	PM-29101	Permian	Gaudalupe Mts., Texas
	Sageceratidae	<i>Pseudosageceras</i> sp.	0.97	1.19	0.31	2.69	0.00	0.14	MM-29104	Triassic	Spitsbergen, Norway
	Paranannitidae	<i>Paranannites aspenensis</i>	1.00	1.07	1.04	1.89	0.19	0.19	MM-29105	Triassic	Crittenden Spring, Nevada
		<i>Paranannites spathi</i>	0.92	1.34	2.16	1.50	0.29	0.17	MM-29106	Triassic	Crittenden Spring, Nevada
		<i>Paranannites</i> sp.	1.01	1.13	1.34	1.76	0.24	0.20	MM-29107	Triassic	Crittenden Spring, Nevada
	Melagathiceratidae	<i>Arnautocelites septentrionalis</i>	1.00	1.15	1.26	1.60	0.23	0.20	MM-29108	Triassic	Crittenden Spring, Nevada
	Arcestidae	<i>Proarcestes gabbi</i>	0.97	1.25	1.17	1.65	0.04	0.17	MM-29109	Triassic	West Humboldt Range, Nevada
	Ptychitidae	<i>Nevadisculites</i> sp.	0.95	1.13	1.88	1.01	0.05	0.15	MM-29110	Triassic	Favret Canyon, Nevada
	Meekoceratidae	<i>Meekoceras gracilitatis</i>	1.03	0.96	0.64	2.50	0.19	0.21	MM-29111	Triassic	Crittenden Spring, Nevada
		<i>Dieneroceras spathi</i>	0.95	1.14	0.59	2.07	0.39	0.23	MM-29112	Triassic	Crittenden Spring, Nevada
		<i>Arctoceras blomstrandii</i>	1.10	1.28	0.81	2.02	0.26	0.22	MM-29113	Triassic	Spitsbergen, Norway
	Ceratitidae	<i>Paraceratites cricki</i>	1.09	0.78	0.68	1.90	0.28	0.23	MM-29114	Triassic	South Tobin Range, Nevada
	Beyrichitidae	<i>Frechites</i> sp.	1.07	0.93	1.17	1.97	0.35	0.25	MM-29115	Triassic	West Humboldt Range, Nevada
	Proteusitidae	<i>Tropigastrites lahontanus</i>	1.09	0.96	1.31	1.47	0.52	0.25	MM-29116	Triassic	South Tobin Range, Nevada
		<i>Favreticeras wallacei</i>	1.09	0.90	0.52	2.21	0.13	0.19	MM-29117	Triassic	McCoy Mine, Nevada
	Proteusitidae	<i>Tropigastrites louderbacki</i>	1.00	0.84	0.86	1.54	0.52	0.24	MM-29118	Triassic	West Humboldt Range, Nevada
		<i>Stolleyites tenuis</i>	1.00	1.10	0.45	1.65	0.00	0.13	MM-29119	Triassic	Spitsbergen, Norway
	Tropitidae	<i>Pleurotropites</i> sp.	0.97	1.12	1.33	1.93	0.04	0.20	MM-29120	Triassic	Block Mts., California
Ammonitida	Phylloceratidae	<i>Phylloceras consanguineum</i>	1.06	1.02	0.52	2.47	0.09	0.19	MM-29121	Jurassic	Sakaraha, Madagascar
		<i>Phylloceras</i> sp.	1.04	1.06	0.64	2.41	0.07	0.19	MM-29122	Cretaceous	Mahajang, Madagascar
		<i>Holcophylloceras</i> sp.	1.09	0.96	0.64	2.18	0.15	0.20	MM-29123	Jurassic	Sakaraha, Madagascar
		<i>Calliphylloceras</i> sp.	1.04	1.00	0.66	1.74	0.13	0.18	MM-29124	Jurassic	Sakaraha, Madagascar
		<i>Ptychophylloceras</i> sp.	1.04	1.01	0.78	2.33	0.04	0.20	MM-29125	Jurassic	Sakaraha, Madagascar
		<i>Phyllopachyceras ezoense</i>	1.08	1.00	0.98	2.94	0.03	0.21	MM-29126	Cretaceous	Saku, Hokkaido, Japan
			1.04	0.96	1.00	2.61	0.01	0.20	MM-29127		
	<i>Hypophylloceras subramosum</i>	1.08	0.94	0.62	2.81	0.07	0.20	MM-29128	Cretaceous	Kotanbetsu, Hokkaido, Japan	
	Ussuritidae	<i>Indigirophyllites spetsbergensis</i>	1.06	0.84	0.77	2.47	0.26	0.25	MM-29129	Triassic	Spitsbergen, Norway
	Juraphyllitidae	<i>Tragophylloceras ibex</i>	0.95	1.01	0.47	1.62	0.24	0.20	MM-29130	Jurassic	Osuarbrück, Germany
	Lytoceratidae	<i>Pterolytoceras</i> sp.	1.07	0.72	1.03	2.17	0.42	0.28	MM-29131	Jurassic	Sakaraha, Madagascar
		<i>Argonauticeras</i> sp.	1.08	0.79	1.16	1.99	0.41	0.28	MM-29132	Cretaceous	Mahajang, Madagascar
	Tetragonitidae	<i>Tetragonites glabrus</i>	1.07	0.97	1.16	2.14	0.32	0.24	MM-29133	Cretaceous	Tappu, Hokkaido, Japan
			1.08	0.91	1.04	2.72	0.20	0.24	MM-29134		
		<i>Tetragonites popetensis</i>	1.10	0.92	0.83	1.86	0.30	0.23	MM-29135	Cretaceous	Saku, Hokkaido, Japan
		<i>Eotetragonites</i> sp.	1.04	0.93	1.31	1.86	0.46	0.25	MM-29136	Cretaceous	Mahajang, Madagascar
	Gaudryceratidae	<i>Gaudryceras striatum</i>	1.07	0.94	1.03	1.99	0.33	0.24	MM-29137	Cretaceous	Saku, Hokkaido, Japan
		<i>Gaudryceras tenuiliratum</i>	1.03	1.00	1.23	1.87	0.44	0.25	MM-29138	Cretaceous	Saku, Hokkaido, Japan
		<i>Gaudryceras denseplicatum</i>	1.10	0.87	0.87	2.22	0.26	0.25	MM-29139	Cretaceous	Tappu, Hokkaido, Japan
		<i>Anagaudryceras limatum</i>	0.92	1.12	1.04	2.01	0.32	0.25	MM-29140	Cretaceous	Kotanbetsu, Hokkaido, Japan
Scaphitidae	<i>Yezoites planus</i>	1.02	1.02	1.27	2.73	0.30	0.24	MM-29141	Cretaceous	Tappu, Hokkaido, Japan	
Douvilleiceratidae	<i>Douvilleiceras</i> sp.	1.05	0.88	0.91	2.20	0.37	0.26	MM-29142	Cretaceous	Mahajang, Madagascar	
	<i>Hypacanthoplites subcornuerianus</i>	1.07	0.80	1.48	2.31	0.32	0.26	MM-29143	Cretaceous	Miyako, Iwate, Japan	
Psiloceratidae	<i>Psiloceras pacificum</i>	0.97	0.91	0.84	1.93	0.39	0.24	MM-29144	Jurassic	New York Canyon, Nevada	
Arietitidae	<i>Agassiceras sciplonianum</i>	1.06	0.71	1.21	2.09	0.47	0.26	MM-29145	Jurassic	Salzgitter, German	
Hildoceratidae	<i>Harpoceras exavatum</i>	1.07	1.00	0.59	2.16	0.23	0.22	MM-29146	Jurassic	Lünde Kanal, Germany	
	<i>Grammoceras doertense</i>	1.26	1.00	0.66	1.75	0.24	0.22	MM-29147	Jurassic	Döruten, Germany	
Graphoceratidae	<i>Leioceras opalinum</i>	1.15	0.83	0.54	2.23	0.22	0.21	MM-29148	Jurassic	Göppingen, Germany	

Order	Family	Species	$a_e$	$a_t$	S	$W_c$	D	$A^{0.5}/L$	Specimens	Age	Locality
Ammonitida	Oppeliidae	<i>Taramelliceras</i> sp.	1.03	0.95	0.52	2.73	0.09	0.20	MM-29149	Jurassic	Sakaraha, Madagascar
		<i>Lissoceras</i> sp.	1.02	0.90	0.69	2.62	0.26	0.24	MM-29150	Jurassic	Sakaraha, Madagascar
	Perisphinctidae	<i>Kranaosphinctes</i> sp.	0.95	0.87	1.08	2.02	0.49	0.26	MM-29151	Jurassic	Sakaraha, Madagascar
		<i>Grossouvia</i> sp.	1.02	0.81	0.82	2.21	0.41	0.26	MM-29152	Jurassic	Sakaraha, Madagascar
	Aspidoceratidae	<i>Aspidoceras</i> sp.	1.03	0.92	1.48	2.09	0.28	0.24	MM-29153	Jurassic	Sakaraha, Madagascar
		<i>Euaspidoceras</i> sp.	0.99	0.61	0.77	2.70	0.37	0.27	MM-29154	Jurassic	Sakaraha, Madagascar
	Craspeditidae	<i>Craspedites subditus</i>	1.03	1.10	1.10	1.68	0.28	0.19	MM-29155	Jurassic	Volga River, Russia
	Desmoceratidae	<i>Desmoceras</i> sp.	0.99	0.99	1.15	2.22	0.20	0.22	MM-29156	Cretaceous	Mahajang, Madagascar
		<i>Tragodesmoceroides subcostatus</i>	1.04	0.98	0.91	2.23	0.11	0.20	MM-29157	Cretaceous	Tappu, Hokkaido, Japan
		<i>Damesites sugata</i>	1.03	1.02	0.85	1.99	0.09	0.19	MM-29158	Cretaceous	Saku, Hokkaido, Japan
		<i>Damesites semicostatus</i>	1.04	0.96	0.90	2.37	0.07	0.21	MM-29159	Cretaceous	Tappu, Hokkaido, Japan
		<i>Damesites</i> sp.	1.07	0.98	0.86	2.05	0.12	0.20	MM-29160	Cretaceous	Saku, Hokkaido, Japan
		<i>Hauericeras angustum</i>	1.06	0.73	0.68	2.41	0.40	0.24	MM-29161	Cretaceous	Saku, Hokkaido, Japan
	Puzosidae	<i>Puzosia</i> sp.	1.01	0.98	1.14	2.13	0.36	0.25	MM-29162	Cretaceous	Mahajang, Madagascar
	Silesitidae	<i>Neosilesites</i> sp.	0.96	0.70	0.82	1.94	0.47	0.25	MM-29163	Cretaceous	Mahajang, Madagascar
	Kossmaticeratidae	<i>Yokoyamaoceras ishikawai</i>	1.15	0.78	0.84	2.33	0.32	0.23	MM-29164	Cretaceous	Saku, Hokkaido, Japan
			1.05	0.97	0.82	1.91	0.33	0.24	MM-29165		
			1.00	0.89	0.90	1.99	0.34	0.23	MM-29166		
	Cleoniceratidae	<i>Cleoniceras</i> sp.	1.00	0.97	0.62	2.62	0.20	0.21	MM-29167	Cretaceous	Mahajang, Madagascar
	Pachydisidae	<i>Anapachydiscus naumanni</i>	1.05	0.92	1.40	2.09	0.24	0.23	MM-29168	Cretaceous	Saku, Hokkaido, Japan
<i>Teshioites ryugasensis</i>			1.02	0.95	0.91	2.08	0.25	0.22	MM-29169	Cretaceous	Saku, Hokkaido, Japan
<i>Canadoceras kosmatti</i>			1.00	1.00	1.14	2.13	0.27	0.23	MM-29170	Cretaceous	Saku, Hokkaido, Japan
Hoplitidae	<i>Neogastrolites meulleri</i>	1.04	0.98	0.66	2.65	0.13	0.21	MM-29171	Cretaceous	Petroleum Country, Montana	